INFLUENCE OF A MAGNETIC FIELD ON THE TAYLOR INSTABILITY IN MAGNETIC FLUIDS

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The magnetization vector deviates from the equilibrium orientation directed along a magnetic field during magnetic fluid motion in the field. A whole number of hydrodynamic effects which are examined systematically in [1], e.g., is due to the interaction between the nonequilibrium component of the magnetization and the external magnetic field. In particular, this mechanism exerts substantial influence on the hydrodynamic stability of shear flows. Thus, it is shown in [2, 3] that there are specific hydrodynamic instabilities in plane-parallel Poiseuille and Couette flows in a homogeneous magnetic field, whose governing parameters are the field magnitude and orientation.

The influence of a magnetic field on the stability of Couette flow between rotating cylinders is investigated in this paper in the narrow gap approximation. The governing mechanism of the instability is the classical Taylor mechanism. The selection of the field orientation in the formulation of the problem is due to the structural features of magnetic-fluid seals of rotating shafts, where Couette flow is realized in a narrow gap. To a significant degree the expansion of the domain of magnetic fluid sealers is related to the rise in the ultimately achievable values of the shaft rotation velocities. However, construction of high-speed seals is a complex engineering problem. Such phenomena as magnetic fluid ejection from the gap being sealed by centrifugal forces, heating of the liquid by dissipative heat liberation, constrain the allowable velocities of shaft rotation [4]. Moreover, in the opinion of the authors of [5], instabilities of circular fluid motion in the gap, particularly the Taylor instability, exert influence on the seal characteristics. According to classical theory, Taylor vortices originate when the modified Reynolds numbers reaches 41.17 [6]. For a gap width $\Delta r \sim 0.2$ mm, shaft diameter $r \sim 20$ mm, and kinematic viscosity coefficient $\eta/\rho \sim 3\cdot 10^{-5}$ m/sec^2 the linear velocity of shaft surface motion is of the order of 60 m/sec at the time of instability origination. Dissipative heating reduces the threshold Reynolds number [7] and the threshold value of the linear velocity for the estimate presented above to 25 m/sec. Crisis changes in the friction moment and power losses are observed in experiments [5] for velocities of such an order. In this connection, a study of the stability of the Couette flow of a magnetic fluid is of practical interest.

The equation of motion of an incompressible magnetic fluid has the form [1, 8]

$$\rho\dot{\mathbf{v}} = -\nabla p + \eta\nabla^2\mathbf{v} + \mu_0 M_0\nabla H + \mathbf{F}. \tag{1}$$

Here H is the field intensity, $M_0(H, T)$ is the equilibrium magnetization,

$$\mathbf{F} = \mu_0\nabla\times(\mathbf{M}'\times\mathbf{H})/2 + \mu_0(\mathbf{M}'\cdot\nabla)\mathbf{H} \tag{2}$$

is the bulk force due to dynamic interaction between the fluid and the field. The deviation of the magnetization **M** from the equilibrium value $M_0\mathbf{e}$ ($\mathbf{e} = \mathbf{H}/H$) is determined by the equation

$$\mathbf{M}' = \mathbf{M} - M_0\mathbf{e} = -\varkappa_\parallel\dot{H}\mathbf{e} - \varkappa_\perp H(\dot{\mathbf{e}} - \mathbf{\Omega}\times\mathbf{e}), \tag{3}$$

where $\mathbf{\Omega} = \nabla\times\mathbf{v}/2$; $\varkappa_\parallel = \varkappa_\parallel(H, T)$, $\varkappa_\perp = \varkappa_\perp(H, T)$ are coefficients of dynamic susceptibility of the magnetic fluid. Equations (1)-(3) must be supplemented by the continuity and magnetostatics equations

$$\nabla\cdot\mathbf{v} = 0,\ \nabla\times\mathbf{H} = 0,\ \nabla\cdot\mathbf{B} = 0,\ \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \tag{4}$$

We write the boundary conditions in the form

$$\mathbf{v}\big|_{r=r_1} = v_1\,\mathbf{i}_\varphi = \omega_1 r_1\,\mathbf{i}_\varphi,\quad \mathbf{v}\big|_{r=r_2} = v_2\,\mathbf{i}_\varphi = \omega_2 r_2\,\mathbf{i}_\varphi, \tag{5}$$

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where r is the radial coordinate, $\mathbf{i}_\varphi$ is the azimuthal direction of the cylindrical coordinate system. The magnetic field in the gap possesses rotational symmetry: At any point it has just r and z components whose distribution is independent of the azimuthal coordinate φ. Since the magnetic flux has a radial direction, we take as the simplest idealization that the magnetic field is directed just along the radius. The field distribution satisfying the Maxwell equations (4) when the fields induced by the fluid are neglected is written in this case in the form $H_r = H_{1r}r_1/r$, where H_{1r} is the field near the inner cylinder. We also provide for the possibility of the existence of a homogeneous field directed along the cylinder axis, which permits comparing the representation of the influence on seal dynamics to the z component of the field. Therefore, within the framework of the assumptions made, the field in (1)-(3) should be considered known

$$\mathbf{H} = H_{1r}(r_1/r)\mathbf{i}_r + H_z\mathbf{i}_z. \tag{6}$$

Here $\mathbf{i}_r$, $\mathbf{i}_z$ are the unit direction of the cylindrical coordinate system. By virtue of the rotational symmetry of the field and the boundary conditions, fluid motion with rotational symmetry can be considered. The simplest motion of this kind is circular $v_\varphi^0 = v^0(r)$. In this case the bulk force (2) has only a φ-component which, taking account of the field distribution (6), we write as

$$F_\varphi = -\frac{\mu_0}{2}\frac{\partial}{\partial r}(\mathbf{M}\times\mathbf{H})_z - \frac{\mu_0}{r}(\mathbf{M}\times\mathbf{H})_z,$$

where $\mu_0(\mathbf{M}\times\mathbf{H})_z = 2\eta_r e_r^2\left(\frac{\partial v^0}{\partial r} - \frac{v^0}{r}\right)$; $\eta_r = \frac{1}{4}\mu_0\varkappa_\perp H^2$ is the rotational viscosity coefficient. Taking account of these relationships, the projections of (1) have the form

$$\rho\frac{\partial v^0}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\eta_e^0\left(\frac{\partial v^0}{\partial r} - \frac{v^0}{r}\right)\right]; \tag{7}$$

$$\frac{\partial p}{\partial r} = \frac{1}{r}\rho v^{02} - \frac{1}{r^2}\mu_0 M_0 H_1 r_1, \tag{8}$$

where

$$\eta_e^0 = \eta + \eta_r e_r^2.$$

Therefore, the influence of the magnetic field on the circular fluid motion reduces to redetermining the viscosity coefficient. The effective viscosity η_e^0 varies along the gap width since the r-component of the unit vector in the field direction and the field magnitude vary

$$e_r = H_{1r}r_1/Hr, \quad H = \sqrt{H_{1r}^2 r_1^2/r^2 + H_z^2}.$$

As follows from (7), the steady flow profile is described by the equation

$$\frac{v^0}{r} = B\int\frac{dr}{\eta_e^0 r^3} + A, \tag{9}$$

in which the constants A and B are determined from the boundary conditions (5). The integral taken in (9) is difficult in the general case because of the complex dependence of the effective viscosity on the independent variable r, however, for certain particular cases (9) can be represented in final form. If the "strong" field condition ($H \gg 15$ kA/m) is satisfied at any point of the gap, the rotational viscosity reaches saturation and is constant in the whole fluid volume $\eta_r = \eta_{rs} = \text{const}$. In this case a flow with the profile

$$\frac{v^0}{r} = \omega_1 + (\omega_2 - \omega_1)\frac{1 - r_1^2/r^2 + Sb\left[\ln(1+b) - \ln\left(r_1^2/r^2 + b\right)\right]}{1 - a^2 + Sb\left[\ln(1+b) - \ln(a^2 + b)\right]}$$

is realized in the gap, where $a = r_1/r_2$; $b = H_z^2/\left[H_{1r}^2(1+S)\right]$; $S = \eta_{rs}/\eta$.

In "weak" fields ($H \ll 15$ kA/m) the constant is the coefficient of dynamic susceptibility. Taking this condition into account $\varkappa_\perp = \varkappa_0 = \text{const}$

$$\frac{v^0}{r} = \omega_1 + (\omega_2 - \omega_1)\frac{\ln(1+S) - \ln\left(1 + Sr_1^2/r^2\right)}{\ln(1+S) - \ln(1 + Sa^2)}, \quad S = \mu_0\varkappa_0 H_{1r}^2/4\eta.$$

When $\Delta r = r_2 - r_1 \ll r$, the change in field magnitude, and therefore, the rotational viscosity in the gap also can be neglected. In a "narrow" gap, therefore a plane-parallel Couette flow

$$v^0 = (v_1 + v_2)/2 + (v_2 - v_1)(r - r_*)/\Delta r, \quad r_* = (r_1 + r_2)/2 \tag{10}$$

is realized for any field magnitude.

The origination of more complex fluid flows with rotational symmetry of the Taylor vortex type is possible for certain cylinder rotation velocities in the magnetic fluid. Let us clarify the influence of the field on this process. We consider the development of small perturbations v(r, z) on the circular motion background. Substituting the solution in the form of the superposition $v^0\mathbf{i}_\varphi + \mathbf{v}$ into the system (1)-(3), after a standard linearization procedure we obtain a system of equations for the small perturbations

$$\rho\left[\frac{\partial v}{\partial t} + \left(\frac{v^0}{r} + \frac{\partial v^0}{\partial r}\right)v_r\right] = \eta\left(\nabla^2 v - \frac{v}{r^2}\right) + F_\varphi; \tag{11}$$

$$\rho\left(\frac{\partial \mathbf{v}_\perp}{\partial t} - \frac{2v^0 v}{r}\mathbf{i}_r\right) = -\nabla p + \eta\nabla^2\mathbf{v}_\perp + \mathbf{F}_\perp, \tag{12}$$

which are, respectively, the φ-projection and the projection on the r–z plane of the equation of motion for the perturbations $\mathbf{v}_\perp = v_r\mathbf{i}_r + v_z\mathbf{i}_z, v \equiv v_\varphi$. Applying the operation ∇× to (12), we eliminate the pressure

$$\rho\frac{\partial}{\partial t}(\nabla \times \mathbf{v}_\perp) + \frac{\partial}{\partial z}\left(\frac{2v^0 v}{r}\right)\mathbf{i}_\varphi = \nabla^2(\nabla \times \mathbf{v}_\perp) + \nabla \times \mathbf{F}_\perp. \tag{13}$$

Since $\mathbf{v}_\perp$ satisfies the continuity equation, it can be expressed in terms of the vector potential $\mathbf{v}_\perp = \nabla \times (\psi\mathbf{i}_\varphi)$. Therefore

$$\nabla \times \mathbf{v}_\perp = \left(-\nabla^2\psi + \frac{\psi}{r^2}\right)\mathbf{i}_\varphi, \quad v_r = -\frac{\partial\psi}{\partial z}, \quad v_z = \frac{1}{r}\frac{d}{dr}(r\psi). \tag{14}$$

The "narrow" gap condition is satisfied in seals; consequently, in what follows we limit ourselves to the examination of just this case. The sources due to the field in (11) and (13) equal to the accuracy of terms on the order of $\Delta r/r_*$

$$F_\varphi = \eta_r(\mathbf{e}\cdot\nabla)^2 v = e_r\frac{\partial^2 v}{\partial r} + 2e_r e_z\frac{\partial^2 v}{\partial r\partial z} + e_z^2\frac{\partial^2 v}{\partial z^2}, \quad \nabla \times \mathbf{F}_\perp = -\eta_r\nabla^4\psi\,\mathbf{i}_\varphi,$$

where η_r, e_r, e_z are constants. Taking these relationships into account, the system (11), (13), and (14) takes the following form in the same approximation:

$$\rho\left(\frac{\partial v}{\partial t} - \frac{\partial v^0}{\partial r}\frac{\partial\psi}{\partial z}\right) = \eta\nabla^2 v + \eta_r(\mathbf{e}\cdot\nabla)^2 v; \tag{15}$$

$$\rho\frac{\partial}{\partial t}\nabla^4\psi - \frac{2v^0}{r_*}\frac{\partial v}{\partial z} = (\eta + \eta_r)\nabla^4\psi; \tag{16}$$

$$v_r = -\frac{\partial\psi}{\partial z}, \quad v_z = \frac{\partial\psi}{\partial r}. \tag{17}$$

The profile of the unperturbed motion v^0 is here determined by the relationship (10). The boundary conditions follow from the condition of disappearance of the perturbations on the layer boundaries

$$\left.\frac{\partial\psi}{\partial z}\right|_{r=r_1,r_2} = \left.\frac{\partial\psi}{\partial r}\right|_{r=r_1,r_2} = \left.v\right|_{r=r_1,r_2} = 0. \tag{18}$$

As is seen from (15) and (16), the dynamical interaction results in anisotropy of the viscous forces in this case. The rotational viscosity affects the r–z- and azimuthal perturbation components differently.

We seek the solution of the system (15), (16) in the form of normal periodic perturbations along the cylinder axis

$$\{v, \psi\} = \{v_a(r), \psi_a(r)\}\exp(ikz - \sigma t).$$

An eigenvalue problem follows for the perturbation amplitudes, and is reduced to dimensionless form. We take as measurement units the gap halfwidth $\Delta r/2$ for the distance, the arithmetic average of the cylinder velocities $(v_2 - v_1)/2$ for the azimuthal velocity component, η/ρ for the stream function, and $\rho\Delta r^2/4\eta$ for the time. As a result we obtain

$$-\sigma v_a - ik\psi_a = \nabla^2 v_a + S(\mathbf{e}\cdot\nabla)^2 v_a; \tag{19}$$

$$-\sigma\nabla^2\psi_a - ik\left[(\mathrm{Re}_2^2 - \mathrm{Re}_1^2) + (\mathrm{Re}_2 - \mathrm{Re}_1)^2 y\right] v_a/16 = (1+S)\nabla^4\psi_a, \tag{20}$$

where

$$S = \frac{\eta_r}{\eta}; \quad \mathrm{Re}_1 = \frac{\rho v_1 \Delta r}{\eta}\sqrt{\frac{\Delta r}{r_*}}; \quad \mathrm{Re}_2 = \frac{\rho v_2 \Delta r}{\eta}\sqrt{\frac{\Delta r}{r_*}}; \quad \nabla^2 = \frac{d^2}{dy^2} - k^2;$$

$$(\mathbf{e}\cdot\nabla)^2 = e_r^2\frac{d^2}{dy^2} + 2ike_r e_z\frac{d}{dy} - k^2 e_z^2; \quad y = \frac{2(r - r_*)}{\Delta r};$$

the same notation is used for the dimensionless variables, except for the radial coordinate, also for the dimensional variables. The origin of the dimensionless radial coordinate y is placed at the middle of the layer. In this case the boundary conditions (18) take the form

$$v = \psi = d\psi/dy = 0 \text{ for } y = \pm 1. \tag{21}$$

The influence of the magnetic field on the eigenvalue spectrum σ of the problem (19)-(21) is governed by the parameter S, which depends on the field magnitude and the field orientation $\alpha = \arctan(e_r/e_z)$ relative to the cylinder axis. In the absence of a field (S = 0) the problem describes the behavior of perturbations in an ordinary viscous fluid.

The problem was solved by the Galerkin method. The spectrum of two-dimensional perturbations in a resting plane-parallel layer of ordinary fluid was chosen as the set of basis functions. If we set $\mathrm{Re}_1 = \mathrm{Re}_2 = S = 0$ in (19) and (20), then the equations

$$\nabla^2 v^{(0)} = -\nu v^{(0)}, \quad \nabla^4\psi^{(0)} = -\mu\psi^{(0)}. \tag{22}$$

follow for its determination. The normalized basis functions satisfy the orthogonality conditions

$$\langle v_n^{(0)} v_l^{(0)}\rangle = \delta_{nl}, \quad \langle\psi_j^{(0)}\nabla^2\psi_m^{(0)}\rangle = -\delta_{jm},$$

where δ_{nl} is the Kronecker delta, and $\langle(\ldots)\rangle = \int_{-1}^{1}(\ldots)\,dy$. The subscripts take on positive integer values corresponding to the eigenvalues ν_i, μ_i numbered in increasing order. The basis defined by (21) and (22) is used extensively to solve viscous fluid dynamics problems [9], as well as ferrohydrodynamics problems [3, 2], and is presented in [3], for example. We represent the solution in the form of the series

$$v = \sum_{n=0}^{N-1} A_n v_n^{(0)}, \quad \psi = \sum_{m=0}^{M-1} B_m\psi_m^{(0)}.$$

After substitution of these relationships into (19) and (20), and scalar multiplication of the first equation by $v_l^{(0)}$ and the second by $\psi_j^{(0)}$ and taking account of the orthogonality conditions, we obtain a system of linear homogeneous equations to determine the expansion coefficients A_n, B_m

$$\begin{gathered}\left[\nu_l(1 + Se_r^2) + Sk^2(e_z^2 - e_r^2) - \sigma\right]A_l - 2ike_r e_z\sum_n\langle v_n^{(0)\prime} v_l\rangle A_n - ik\sum_m\langle\psi_m v_l\rangle B_m = 0,\\ \sum_n\langle\left[(\mathrm{Re}_2^2 - \mathrm{Re}_1^2) + (\mathrm{Re}_2 - \mathrm{Re}_1)y\right]ikv_n\psi_j\rangle + \left[(1+S)\mu_j - \sigma\right]B_j = 0.\end{gathered} \tag{23}$$

The eigenvalues σ of the coefficient matrix of the homogeneous system (23) were determined numerically by the Greenstadt method [10]. The eigenvector corresponding to the eigenvalue with minimal real part was found by the inverse iteration method [11]. Computations were performed for M = N = 4, which yields an error no greater than 3% in the determination of the minimal eigenvalue; the accuracy was determined by comparison with checking computations for M = N = 8, when an increase in the number of basis functions has practically no influence on the lower eigenvalue.

Let us discuss the results of the numerical investigation for the case when the outer cylinder is at rest, i.e., $\mathrm{Re}_2 = 0$. As Re_1 increases, the real part of the eigenvalue that is minimal in absolute value diminishes and changes sign for a certain $\mathrm{Re}_1 = R_1(\alpha, S, k)$, which indicates flow instability. Here if α ≠ 0.90° the imaginary part of the unstable mode is different from zero. Therefore, the Taylor instability is fluctuating in nature in an oblique magnetic field. In purely radial and axial fields (α = 0.90°) the neutral perturbation decrements are real. In this case, the instability is monotonic in nature as in the absence of a field. Neutral curves $R_1(k)$ are presented in Fig. 1 for S and α (the numbers

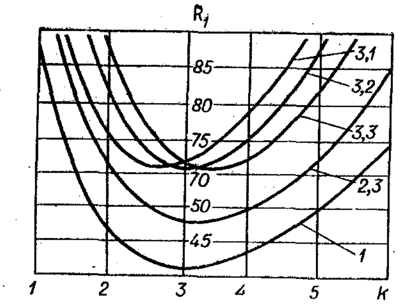

Fig. 1

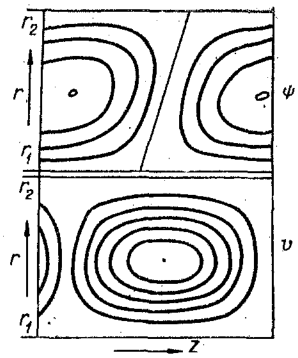

Fig. 2

1-3 on the left correspond to S = 0, 0.2, 1 and on the right α = 0; 45; 90°). The coordinates of the minimum on these curves represent the threshold Reynolds number R_{1*} and the wave number k_* of the threshold perturbation.

Investigation of the influence of the field on the threshold characteristic was carried out in the parameter range $0 < S < 1$, $-90 < \alpha < 90°$. It is seen from Fig. 1 that R_{1*} is practically independent of the field orientation and can be approximated with a high degree of accuracy by the relationship $R_{1*} = 41.17 + 30\ S$, which we represent in the form

$$\tilde{R}_{1*} = \frac{R_{1*}}{1+0.73S} = \frac{\rho v_1 \Delta r}{\eta + 0.73\eta_r}\sqrt{\frac{\Delta r}{r_*}} = 41.17.$$

It hence follows that the threshold instability can be determined by using a renormalized Reynolds number $\tilde{R}_{1*}$ which has a fixed value in a field of arbitrary magnitude and orientation. Renormalization is accomplished by using a certain effective viscosity

$$\eta_e = \eta + 0.73\eta_r \tag{24}$$

that depends only on the field magnitude and is independent of its orientation, instead of the dynamic viscosity coefficient. It differs from the effective viscosity η_e^0 determining the energy dissipation in the gap in the circular flow mode. The dependence of the threshold wave number on the field characteristics can be described by the relationship

$$k_* = 3.18 - 0.5S\cos 2\alpha.$$

If the field direction approaches the axial, then threshold wavelength $\lambda_* = 2\pi/k_*$ increases. The radial field results in a diminution of λ_*. For α = 45° the field does not alter the perturbation wavelength. If the renormalized wavelength $\tilde{k} = k/(1 - 0.16\ S\cos 2\alpha)$ is used, the instability can be described by a universal neutral curve $\tilde{R}_1(\tilde{k})$ whose form is independent of the field characteristics and agrees with the neutral instability curve in an ordinary viscous fluid (curve 1 in Fig. 1).

In an oblique field the neutral perturbations are traveling waves. Their propagation direction is opposite to the direction of the field z-component. The dependence of the frequency of fluctuation of the threshold perturbation is approximated well by the relationship $\mathrm{Im}\,\sigma = -0.16\ S\sin 2\alpha$.

Isoline patterns of the azimuthal velocity component and stream function are represented in Fig. 2 for α = 45° (S = 1, k = 2.5, $Re_1 = 75$, $Re_2 = 0$) when the influence of the field on the perturbation structure is maximal. This influence is examined most clearly in the isoline pattern for v. As is seen, the line separating two convective cells passes obliquely to the layer. In the absence of a field, as well as in purely radial and axial fields, when the vortices being generated in the shear flow are stationary, this line is perpendicular to the layer boundaries.

Let us discuss the results of investigating the influence of the field on the threshold characteristics when both cylinders rotate. Rotation of the outer cylinder is the same direction as the inner does not result in a qualitative change in the structure of the threshold perturbations. The form of the v and ψ isolines is analogous to that represented in Fig. 2. The change in the parameter Re_2 is here also of slight influence on the threshold wavelength and vibration frequency.

When the cylinders rotate in different directions in an ordinary fluid, the Taylor vortices develop in the domain of the gap between the inner cylinder and the fluid layer for

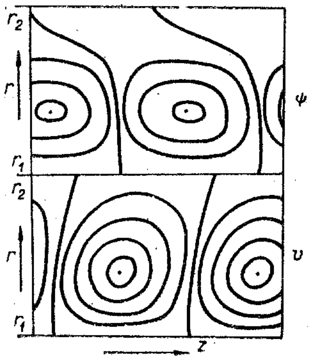

Fig. 3

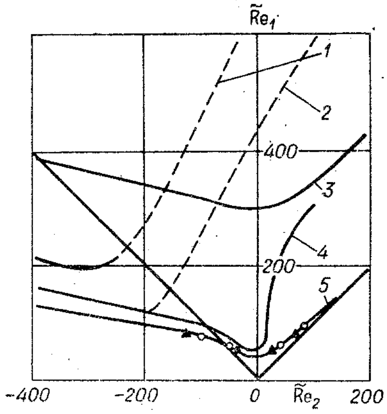

Fig. 4

which $v^0 = 0$. The same vortex structure holds in purely radial and axial fields. As is seen from Fig. 3, where isolines are shown for the case $\alpha = 45°$, $S = 1$, $Re_1 = 110$, $Re_2 = -100$ the vortices in an oblique field penetrate the domain abutting on the outer cylinder.

The dependence of $\tilde{R}_{1*}$ on the renormalized Reynolds number $\tilde{Re}_2 = Re_2/(1 + 0.73S)$ determined by means of the rotational velocity of the outer cylinder is presented in Fig. 4. Curve 5 corresponds to the case $S = 0$. Points for $S = 1$, $\alpha = 45°$ are marked with the shaded triangles ▲. This curve which demarcates the plane $\tilde{Re}_1$-$\tilde{Re}_2$ into two domains where the circular motion is stable (lower part of the plane) and unstable is universal in nature. Its shape is practically independent of the field characteristics.

Therefore, the influence of the magnetic field on the threshold of Taylor instability occurrence in narrow gaps in the investigated range of external parameters can be described by introducing the effective viscosity coefficient defined by the relationship (24) into the modified Reynolds number in place of the dynamical viscosity coefficient. As in an ordinary viscous fluid, the instability in purely radial and axial fields is monotonic in nature, while in an oblique field it is vibrational. It should be noted that there are significant axial field gradients in seals, where the z-component of the field changes sign. This can cause generation of waves being propagated in opposite directions whose addition will result in stationary vortex formation.

Origination of Taylor vortices is the first stage on the road to the passage of circular into turbulent motion as the Reynolds number increases. Because of the universality of the stability pattern of circular motion in the $\tilde{Re}_1$, $\tilde{Re}_2$ axes (see Fig. 4), it is expedient to denote the boundary of the different flow modes thereon. For this we use the data for an ordinary viscous fluid presented in [12]. The symbols O in Fig. 4 mark the $\tilde{Re}$ for $r_1/r_2 = 1.135$. As is seen, for such a relationship between the cylinder radii the data of [12] will lie on the curve 5 obtained in the narrow-gap approximation. Curve 4 is the boundary of secondary wavy Taylor vortex formation, 2 is the boundary of the transition region to turbulent flow, 1 is the boundary of the turbulent flow domain for $r_1/r_2 = 1.135$, and 3 is the boundary of an abrupt change in the friction moment for $r_1/r_2 = 1.176$. As is seen, the development of turbulence and its associated growth of the friction moment starts for Reynolds numbers significantly exceeding the boundary of Taylor vortex formation (for the outer cylinder at rest). The Reynolds numbers achieved in modern seals lie below the boundary of turbulent mode origination.

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NONSTEADY PROPERTIES OF COUETTE FLOW OF A LIQUID UNDER THE CONDITIONS OF A PHASE TRANSITION

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UDC 532.54+532.78

Liquid flow under the conditions of the concurrent interaction of dissipative heat release and a phase transition was investigated in [1, 2]. In this case a quasi-steady approximation with respect to velocity and temperature was used, making it possible to determine the regions of the characteristic flow regimes: a complete phase transition, a regime of steady flow with the phase interface at an intermediate position, and a regime of hydrodynamic thermal explosion (HTE) [3].

Such an approach, presuming a sufficiently great heat of the phase transition and that the initial temperature and velocity distributions belong to the region of attraction of steady-state profiles, has a limited applicability. A clarification of the region of its applicability — the problem of nonsteady analysis — is discussed in the present paper.

1. Statement of the Problem

We consider the Couette flow of a viscous incompressible liquid lying between two coaxial infinite cylinders; the inner one (with a radius r_0) rotates while the outer one (with a radius r_1) is stationary. The outer cylinder is cooled below the temperature T_* of the phase transition, as a result of which a layer of solid material of thickness $\Delta = r_* - r_0$ is formed, where r_* is the coordinate of the phase interface. The Arrhenius temperature dependence of the viscosity $\eta = \eta_0 \exp(E/RT)$ is adopted, where E is the activation energy of the viscous flow, R is the universal gas constant, η_0 is a preexponential factor, and T is the temperature.

The system of equations of heat conduction and motion and the rheological equation can be written in the form

$$r<r_*:\ c_1\rho_1 \frac{\partial T}{\partial t} = \lambda_1\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \sigma r \frac{\partial \Omega}{\partial r}; \quad (1.1)$$

$$\frac{\partial \Omega}{\partial t} = \frac{1}{\rho_1 r^3}\frac{\partial}{\partial r}(\sigma r^2),\quad \sigma = \eta r \frac{\partial \Omega}{\partial r}; \quad (1.2)$$

$$r>r_*:\ c_2\rho_2 \frac{\partial T}{\partial t} = \lambda_2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \quad (1.3)$$

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